CONJUGATE UNSTEADY HEAT TRANSFER IN A LAMINAR BOUNDARY LAYER ON A SEGMENTED FLAT PLATE

V. V. Sapelkin

UDC 536.24.02

In many studies [1-7], heat transfer in flows past solid bodies with a sharp variation in boundary conditions was analyzed with a priori specification of surface temperature or heat flux as a step or sinusoidal function. Factors causing such a variation in boundary conditions were not taken into consideration. Hence it is more comprehensive to consider a problem with heat transfer in the solid body, i.e., as a conjugate problem. Thus, in [8, 9] the conjugate problem considered the effect on body surface temperature and the coefficient of heat transfer in boundary layer with segmented blowing, whereas in [9], the problem was considered nonstationary. The present paper uses the finite-difference method to study the effect of sudden change in thermal properties of the flat plate in the streamwise direction on the unsteady heat transfer characteristics of the surrounding fluid.

1. Formulation of the Problem. At the time just preceding the initial moment ($t < 0$), let the flat plate of length L and thickness 2b have temperature T_0 different from the free stream temperature T_∞. Coordinates x and y are along and perpendicular to the plate. For simplicity, consider that the plate consists of two sections along the flow direction with different thermal characteristics. Ideal thermal contact of segments is assumed. Unsteadiness is due to instantaneous introduction of the plate in the flow at $t = 0$. Since the problem is symmetric relative to the plane $y = -b$, it is sufficient to consider a plate with thickness b, whose outer surface is insulated. The following assumptions are made: incompressible flow — thermal properties of the fluid and the plate do not depend on temperature; laminar flow — the plate is sufficiently thin so that the temperature gradient across the plate is negligible.

Nondimensional equations of the problem have the following form:

$$\Theta_{\mu\mu} + (\Pr \varphi\delta + B\xi\delta\mu\delta_\tau)\Theta_\mu - 2\Pr \xi \varphi_\mu \delta \Theta_\xi = B\xi\delta^2\Theta_\tau; \tag{1.1}$$

$$\Theta = \Theta_0 = 0,\ \tau = 0,\ 0 \leqslant \xi \leqslant 1,\ 0 \leqslant \mu \leqslant 1; \tag{1.2}$$

$$\Theta = \Theta_\infty = 0,\ \tau > 0,\ \xi = 0,\ 0 \leqslant \mu \leqslant 1; \tag{1.3}$$

$$\Theta = \Theta_\infty = 0,\ \tau > 0,\ 0 \leqslant \xi \leqslant 1,\ \mu = 1; \tag{1.4}$$

$$\Theta = \Theta_w(\xi, \tau),\ \tau > 0,\ 0 \leqslant \xi \leqslant 1,\ \mu = 0; \tag{1.5}$$

$$c\rho_w(\xi)\frac{\partial\Theta_w}{\partial\tau} = \frac{\partial}{\partial\xi}\left(\lambda_w(\xi)\frac{\partial\Theta_w}{\partial\xi}\right) + \gamma\xi^{-1/2}\frac{1}{\delta}\left(\frac{\partial\Theta}{\partial\mu}\right)_{\mu=0}; \tag{1.6}$$

$$\lambda_w(\xi)\frac{\partial\Theta_w}{\partial\xi} = \mathrm{Bi}_{\text{н}}\Theta_w,\ \tau > 0,\ \xi = 0; \tag{1.7}$$

$$\lambda_w(\xi)\frac{\partial\Theta_w}{\partial\xi} = \mathrm{Bi}_{\text{к}}\Theta_w,\quad \tau > 0,\quad \xi = 1; \tag{1.8}$$

$$\Theta_w = \Theta_{w0} = 1,\ \tau = 0,\ 0 \leqslant \xi \leqslant 1; \tag{1.9}$$

$$\lambda_w(\xi) = \begin{cases}\lambda_{w1}, & 0 \leqslant \xi \leqslant \xi^*,\\ \lambda_{w2}, & \xi^* < \xi \leqslant 1,\end{cases}\quad c\rho_w(\xi) = \begin{cases}c\rho_{w1}, & 0 \leqslant \xi \leqslant \xi^*,\\ c\rho_{w2}, & \xi^* < \xi \leqslant 1.\end{cases} \tag{1.10}$$

Here $\Phi = (T - T_\infty)/(T_0 - T_\infty)$; $\Theta_w = (T_w - T_\infty)/(T_0 - T_\infty)$; $\xi = x/L$; $\mu = \eta/\delta(\tau)$; $\eta = 0.5y\sqrt{U_\infty/\nu x}$; $\tau = a^*t/L^2$; $\mathrm{Re} = U_\infty L/\nu$; $\Pr = \nu/a_j$; $\gamma = 0.5(L/b)(\lambda_f/\lambda^*)\sqrt{\mathrm{Re}}$; $B = 4(a^*/a_f)\mathrm{Re}^{-1}$; $\mathrm{Bi} = \alpha L/\lambda^*$; $\delta(\tau)$ is the nondimensional thermal boundary-layer thickness determined from the condition of smooth transition of temperature at the outer edge of the boundary layer, which has the fol-

Zhukovskii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 90-95, May-June, 1985. Original article submitted March 11, 1984.

 0021-8944/85/2603-0382$09.50

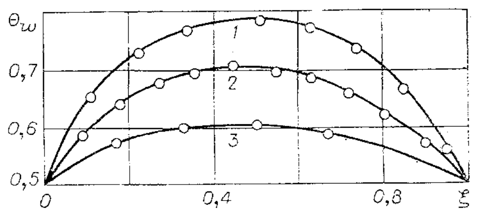

Fig. 1

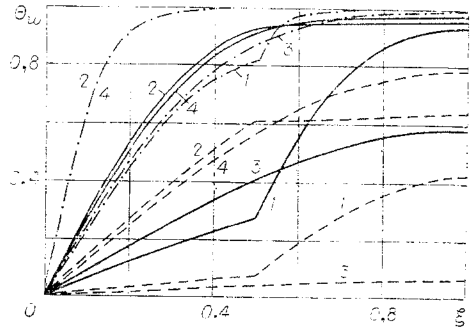

Fig. 2

lowing form

$$\left|\frac{1}{\delta}\frac{\partial\Theta}{\partial\mu}\right|\leqslant\varepsilon,\ 0<\xi\leqslant 1,\mu=1. \tag{1.11}$$

The indices denote the following f, fluid parameters: 0, initial conditions; H and K, parameters when $\xi = 0$ and $\xi = 1$, respectively; w, plate parameters; ∞, free stream parameters; *, reference value of the parameters; 1 and 2, parameters at each section of the plate; η, ξ, τ indicate differentiation with respect to the respective variables. The stream function φ and its derivatives are determined from the solution to the Blasius problem [10]. The variable μ is used to ensure automatic increase in the grid time interval along the coordinate η for a constant number of grid points but in so doing the value of $\delta(\tau)$ and its time derivative should be corrected at each time interval [11].

2. Finite-Difference Equations. The boundary-value problem (1.1)-(1.11) is approximated by the following finite-difference scheme:

$$-A_{m,n}\Theta_{m,n-1}^{j+1}+C_{m,n}\Theta_{m,n}^{j+1}-B_{m,n}\Theta_{m,n+1}^{j+1}=F_{m,n}; \tag{2.1}$$

$$\Theta_{0,n}^{j+1}=\Theta^{(\infty)j+1}=0; \tag{2.2}$$

$$\Theta_{m,N}^{j+1}=\Theta^{(\infty)j+1}=0; \tag{2.3}$$

$$\Theta_{m,0}^{j+1}=\Theta_m^{(w)j+1}; \tag{2.4}$$

$$\Theta_{m,n}^{0}=\Theta_{m,n}^{(0)}=0; \tag{2.5}$$

$$-A_m^{(w)}\Theta_{m-1}^{(w)j+1}+C_m^{(w)}\Theta_m^{(w)j+1}-B_m^{(w)}\Theta_{m+1}^{(w)j+1}+D_m^{(w)}\left(\Theta_{m,1}^{j+1}-\Theta_m^{(w)j+1}\right)=F_m^{(w)}; \tag{2.6}$$

$$\Theta_m^{(w)0}=\Theta_m^{(w0)}=1; \tag{2.7}$$

$$\Theta_0^{(w)j+1}=\varkappa_{\text{H}}\Theta_1^{(w)j+1}+\nu_{\text{H}},\quad 0\leqslant\varkappa_{\text{H}}\leqslant 1; \tag{2.8}$$

$$\Theta_M^{(w)j+1}=\varkappa_{\text{K}}\Theta_{M-1}^{(w)j+1}+\nu_{\text{K}},\quad 0\leqslant\varkappa_{\text{K}}\leqslant 1; \tag{2.9}$$

$$p\Theta_{m,N}^{j+1}+k\Theta_{m,N-1}^{j+1}+l\Theta_{m,N-2}^{j+1}\leqslant\varepsilon\delta^{j+1}. \tag{2.10}$$

The coefficients in Eqs. (2.1) and (2.6) and boundary conditions (2.8)-(2.10) are determined by the requirements (approximation, stability, homogeneity, etc.) of the finite difference scheme. For our purpose, it is convenient to use homogeneous, implicit difference scheme satisfying conservative and monotonic laws.

Indices m, n, and j in Eqs. (2.1)-(2.10) indicate grid points in the ξ, μ, and τ directions, respectively; M = max (m), N = max (n).

3. Solution to Finite-Difference Equations. The system of difference equations (2.1)-(2.10) has such a form that a direct application of the usual marching technique does not appear to be possible to solve and thermodynamic equilibrium equations cannot be solved independently, and, secondly, the initial-value problem is solved along ξ for the energy equation, whereas the boundary-value problem is solved for thermodynamic equilibrium.

In order to solve the system of difference equations of the type (2.1)-(2.10), Sapelkin [12] suggested the method of streamwise-normal marching, whose characteristic feature is the computation of streamwise (along index m) marching parameters using normal (along the index n) marching coefficients.

In order to solve the given problem, a modification of the algorithm given in [12] was used, because the thermal properties of each plate segment could vary over a wide range. In this case, streamwise variant [13] of streamwise marching was used to improve accuracy.

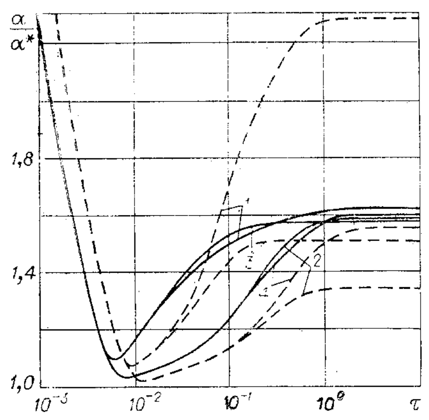

Fig. 3

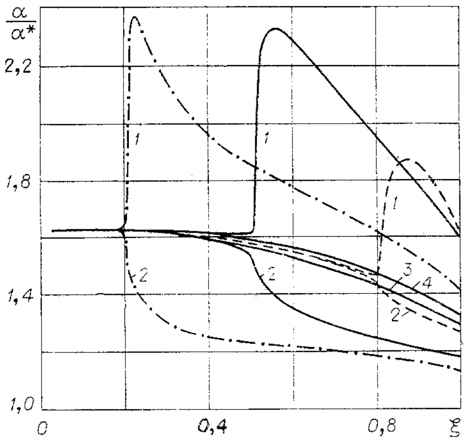

Fig. 4

4. Computational Results. The algorithm was programmed in FORTRAN and series of computations were carried out on a BÉSM-6 computer. Verification of the algorithm and the choice of parameters for the difference scheme were made using the solution to the conjugate problem of steady heat transfer in a laminar boundary layer on a solid flat plate whose surface is maintained at a constant temperature [14]. Figure 1 shows computed results (solid curves) and data from [14] (circles). The following parameters were chosen: Pr = 1.0, $\Theta_w(0) = \Theta_w(1) = 0.5$, $\gamma = 10.0$ (curve 1), $\gamma = 5.0$ (curve 2), $\gamma = 2.0$ (curve 3). It is seen that for a wide range of these conjugate parameter γ results are practically identical; the difference does not exceed 1%.

Computed results for the given boundary-value problem are shown in Figs. 2-4. Computations were carried out for four plates consisting of two segments along the flow: Π_1 made of copper-steel; Π_2, steel-copper; and also for fully copper Π_3 ($\lambda = 395$ W/m·°K, $c\rho = 3.4 \cdot 10^6$ J/m^3·°K) and steel Π_4 ($\lambda = 53$ W/m·°K, $c\rho = 3.95 \cdot 10^6$ J/m^3·°K) plates (curves 1-4). The plate length was L = 1 m, half-thickness b = 0.01 m. The free stream parameters were Re = 1000 and Pr = 0.688. Values corresponding to the properties of copper were taken as reference quantities: B = 0.01375 and $\gamma = 0.1281$. It was assumed that the left cover of the plate instantaneously attains the free stream temperature, $Bi_H = \infty$, and the right segment is heat insulated $Bi_K = 0$.

Figure 2 shows the temperature distribution on the plate along its length at different moments in time: $\tau = 0.0519$ (dashed-dotted line), $\tau = 0.3014$ (solid lines), and $\tau = 1.1821$ (dashed lines). The characteristic feature of the temperature distribution along the segmented plates Π_1 and Π_2 is the break in curves at the point of contact of the segments ($\xi = \xi^* = 0.5$). Analogous curves for solid plates Π_3 and Π_4 are monotonic.

Variation in relative local heat transfer coefficient with time for $\xi = 0.4$ (solid curves) and $\xi = 0.6$ (dashed lines) is shown in Fig. 3. The reference scale is the heat-transfer coefficient at constant surface temperature [10].

$$\alpha^* x/\lambda_f = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{0.33}.$$

Curves for the variation in heat transfer coefficient have a minimum whose value $(\alpha/\alpha^*)_{min}$ and the corresponding time τ_{min} depend on the coordinate ξ and thermal properties of the plate. Such a behavior of the unsteady heat-transfer coefficient was earlier observed theoretically as well as experimentally for free and forced convection in [12, 15-17]. This is associated with the fact that at the initial stages the dominating influence on heat transfer between the fluid and the solid body is due to thermal conductivity (the left branch of curves relative to the minimum) and at large times it is due to forced convection (right branch of the curves) and later still its effect is realized farther downstream of the given point. This explains that with an increase in ξ, there is an increase in τ_{min} but a decrease in $(\alpha/\alpha^*)_{min}$. The value of the minimum heat-transfer coefficient and the corresponding time for each pair of plates Π_1, Π_3 and Π_2, Π_4 are identical, whereas during the transition from the first to the second pair of plates $(\alpha/\alpha^*)_{min}$ decreases and τ_{min} increases marginally. When $\tau > \tau_{min}$, the local heat-transfer coefficient increases monotonically to its steady-state values.

Starting from a certain time (the lower its value, the larger the value of ξ), curves of local heat transfer coefficients for each of the above-mentioned pairs of plates diverge and subsequently the heat-transfer characteristics of each plate are different.

The distribution of steady-state heat-transfer coefficients along the plate length is given in Fig. 4. Steady-state heat-transfer coefficients in the first segment ($\xi \leqslant \xi^*$) of the plates Π_1 and Π_2 are close not only to each other but also to heat-transfer coefficients at the same points of continuous plates Π_3 and Π_4. In the second segment of segmented plates the distributions of heat-transfer coefficients differ from each other and also from the distribution of heat-transfer coefficients in the same segment of the continuous plate. Thus, when $\xi^* = 0.5$ (continuous curves) the heat-transfer coefficient behind the point $\xi = \xi^*$ for the segment plate Π_1 increases rapidly, attains a maximum, and subsequently decreases monotonically. For the segmented plate Π_2, the heat-transfer coefficient decreases rapidly behind the point $\xi = \xi^*$. Analogous distribution of heat transfer coefficients along the length of the segmented plates Π_1 and Π_2 occurs even at other ξ^* ($\xi^* = 0.2$ is indicated by dash-dot lines and $\xi^* = 0.8$ by dashed lines). With an increase in ξ^* the value of the maximum heat transfer coefficient on the plate Π_1 decreases and for the plant Π_2 a slower decrease in heat transfer coefficient is observed.

The above-described nature of the distribution of heat transfer coefficient along the length of the segmented plate is explained by a sharp change in the temperature gradient in the plate at the location of the discontinuity in thermophysical properties. This leads to a restructuring of temperature profiles in the boundary layer downstream of the point where there is a sudden change in the thermophysical properties, expressed by a sharp increase in the heat-transfer coefficient for the plate Π_1 and a decrease for the plate Π_2.

The nature of the steady-state distribution of the heat-transfer coefficient along the surface of the body whose thermophysical properties vary sharply along the flow differs qualitatively from similar distributions for multilayered bodies whose thermophysical properties vary sharply in a direction normal to the flow. In the latter case, as shown in [18], from a solution of the conjugate problem of steady-state heat transfer with transverse flow past a two-layered cylinder, the curve for heat transfer coefficient is located between similar curves obtained for continuous cylinders whose thermophysical properties are identical for each layer. This makes it possible to obtain, on the basis of the solution to the continuous body problem, the upper and lower bounds for the heat transfer coefficient of multilayered bodies, which is not possible according to Fig. 4 for bodies segmented in the direction of flow.

LITERATURE CITED

1. P. M. Moretti and W. M. Kays, "Heat transfer to a turbulent boundary layer with varying free-stream velocity and varying surface temperature. An experimental study," Int. J. Heat Mass Transfer, 8, No. 9 (1965).
2. T. Cebeci, A. M. O. Smith, and G. J. Mosinskis, "Solution to incompressible turbulent boundary layer equations with heat transfer," Trans. ASME, Ser. C, Heat Transfer, No. 2 (1970).
3. T. Cebeci and A. M. O. Smith, Analysis of Turbulent Boundary Layers, Academic Paess, New York (1974).
4. N. P. Kolina, "Effect of step-change in surface temperature on heat transfer in laminar boundary layer. Aerodynamic heating at supersonic speeds," Tr. TsAGI (Proc. of Central Aero-Hydrodynamics Institute) 1315 (1971).
5. V. S. Astavin, I. O. Korolev, and Yu. S. Ryazantsev, "On the temperature in channel flow with a jump in wall temperature," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1979).
6. N. M. Galin and A. I. Groshev, "Influence of streamwise turbulent diffusion on heat-transfer laws in a pipe," Teploenergetika, No. 5 (1979).
7. V. A. Aleksin and V. D. Sovershennyi, "Computation of turbulent boundary layer with sharp variation in boundary conditions," in: Turbulent Flows [in Russian], Nauka, Moscow (1977).
8. D. A. Nealy, "Some effects of variable surface temperature on heat transfer in partially porous flat plate," Trans. ASME, Ser. A, J. Eng. Power, No. 4 (1973).
9. V. I. Zinchenko, E. N. Putyatina, and E. G. Trofimchuk, "Computation of characteristics of conjugate heat transfer with sharp variation in boundary conditions," in: Heat Transfer — VI, Proc. of 6th All-Union Conference on Heat and Mass Transfer [in Russian], Vol. 1, Part 2, Minsk (1980).
10. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1981).

11. V. M. Paskonov, "On an algorithm to solve boundary-layer problems," in: Computational Methods and Programming [in Russian], Vol. 7, Izd. Mosk. Gos. Univ. (1967).
12. V. V. Sapelkin, "Conjugate problem of unsteady heat transfer between laminar boundary layer and flat plate with internal heat source," Teplofiz. Vys. Temp., No. 6 (1981).
13. A. A. Samarskii and E. S. Nikolaev, Methods of Solving Finite-Difference Equations [in Russian], Nauka, Moscow (1978).
14. D. I. Didenko, "Computation of heat transfer between a flat plate heated at the ends and the surrounding fluid flow," Teplofiz. Teplotekh., 23 (1973).
15. M. Soliman and H. A. Johnson, "Transient heat transfer for turbulent flow over a flat plate of appreciable thermal capacity and containing time-dependent heat source," Trans. ASME, Ser. C, J. Heat Transfer, 89, No. 4 (1967).
16. R. C. C. Wang, B. T. F. Chung, and L. C. Thomas, "Transient convective heat transfer for laminar boundary layer flow with effects of wall capacitance and resistance," Trans. ASME, Ser. C, J. Heat Transfer, 99, No. 4 (1977).
17. R. G. Kemink and E. M. Sparrow, "Heat transfer coefficients for melting about a vertical cylinder with or without subcooling and for open or closed containment," Int. J. Heat Mass Transfer, 24, No. 10 (1981).
18. B. Sunden, "Conjugated heat transfer from circular cylinders in low Reynolds number flow," Int. J. Heat Mass Transfer, 23, No. 10 (1980).

FLOW STRUCTURE NEAR THE TRAILING EDGE OF A PLATE

V. V. Bogolepov

UDC 532.526.011:518.5

Solutions were obtained in [1-3] for the vicinity of the trailing edge of a plane plate at high but precritical Reynolds numbers Re_0, calculated with the plate length l and incident flow parameters, for subsonic and supersonic external flows, which describe the motion in a transition region of extent $x \sim O(l\, Re_0^{-3/8})$ between the known Blasius flow on the plane plate and the flow in the wake [4]. These solutions have a singularity in the wake behind the plate, which can be overcome by the use of numerical methods. The presence of this singularity indicates the need to study the flow in the region $x < l\, Re_0^{-3/8}$.

The present study will use the method of combined asymptotic expansions as $Re_0 \to \infty$ to study the flow near the trailing edge of a plate within the region $lRe_0^{-3/4} < x < lRe_0^{-3/8}$. It is found that at such lengths in the region near the plate a "compensation" flow regime is realized [5], wherein the solutions of [1-3] are valid for the rear edge of the plate, and a singularity of the former type exists in the wake. It is shown that in the singular region at $x \sim O(lRe_0^{-3/4})$, in a first approximation the flow may be described by the Navier–Stokes equations for an incompressible liquid. Numerical solutions are obtained for a thin plate and a thick plate over a wide range of local Reynolds number Re = 0-100. Flow line patterns, detachment zone characteristics, and gas dynamic flow function distributions over the surface of the bodies are presented.

1. In constructing the solutions of [1-3] to evaluate flow functions in the narrow region near the surface of the plate, it was assumed that the flow functions change in proportion to distance from the plate surface, that the flow was viscous, and that the discontinuity in boundary conditions at the trailing edge of the plate produced nonlinear perturbations of the flow functions. Then, using the equations of motion of the liquid, we easily obtain

$$u \sim x^{1/3},\ v \sim \varepsilon x^{-1/3},\ \Delta p \sim x^{2/3},\ \delta \sim \varepsilon x^{1/3}. \tag{1.1}$$

Here and below we will use dimensionless variables; for this purpose all linear dimensions are referred to l, pressure and enthalpy to $\rho_0 u_0^2$ and u_0^2, respectively; the remaining flow functions are referred to their values in the unperturbed incident flow; δ is the thickness of the mixing layer behind the plate edge; $\varepsilon = Re_0^{-1/2}$.

In the flow under consideration the origin of mixing layer formation x = 0 is fixed, and so Eq. (1.1) describes a singularity immediately behind the trailing edge of the plate. Equation (1.1) is complemented by the conditions of interaction in the layer near the plate

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 95-99, May-June, 1985. Original article submitted April 3, 1984.

 0021-8944/85/2603-0386$09.50